HIGGS PRODUCTION AT N3LO

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based on work in collaboration with:
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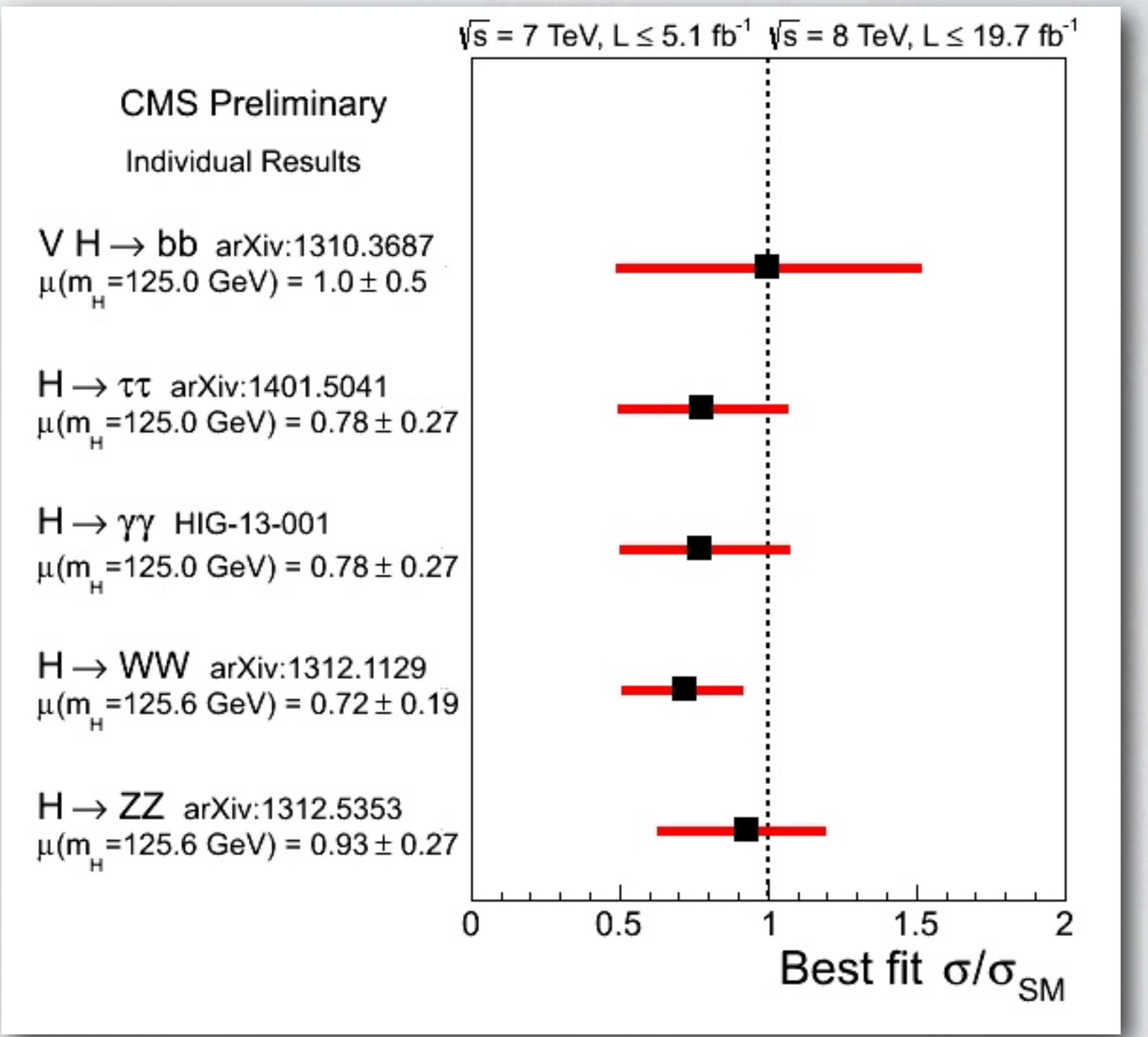
Motivation

- Discovery marks the beginning of the experimental era of Higgs physics

- Determination of the properties of the Higgs will be a challenge for years to come

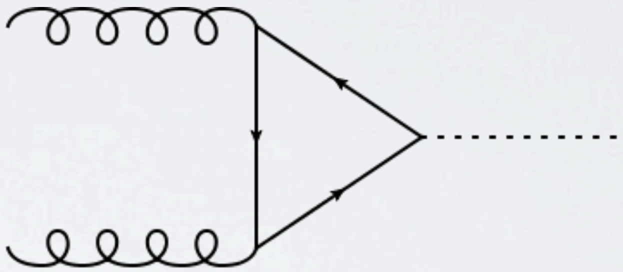
- Requires precision measurements and predictions



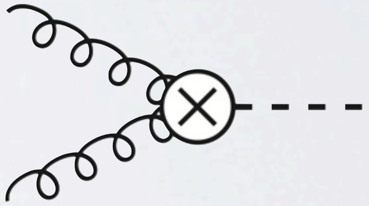
Amazing progress form the experiments

The gluon fusion cross section

- The dominant Higgs production mode at the LHC is gluon fusion
    - Loop-induced process



- The Higgs boson is light compared to the top quark
- The top loop can be integrated out → effective theory



- The tree-level coupling of the gluons to the Higgs is described by a dimension five operator

$$\mathcal{L} = \mathcal{L}_{\mathrm{QCD}} - \frac{1}{4v} C_1 H G^a_{\mu\nu} G_a^{\mu\nu}$$

The gluon fusion cross section

- Operators with higher dimension can be included in the computation
- This leads to a systematic expansion of the gluon fusion cross section in the top mass
- Sub-leading corrections in the top-mass are known at NNLO
  [Harlander, Ozeren; Pak, Rogal, Steinhauser; Ball, Del Duca, Marzani, Forte, Vicini; Harlander, Mantler, Marzani, Ozeren]
- In the following I will only talk about the leading term in the effective theory

The gluon fusion cross section

- The gluon fusion cross-section in perturbation theory is

$$\sigma\left(pp \to H + X\right) = \tau \sum_{ij} \int_{\tau}^{1} dz \mathcal{L}_{ij}(z) \hat{\sigma}_{ij}\left(\frac{\tau}{z}\right)$$

- We compute the inclusive partonic cross section

- The partonic cross section is a function of

$$z = \frac{m_h^2}{\hat{s}} \quad \to \quad \bar{z} = \frac{\hat{s} - m_h^2}{\hat{s}} \qquad\qquad \tau = \frac{m_h^2}{E_{cm}^2}$$

- In perturbation theory the partonic cross section can be expanded

$$\hat{\sigma}(z) = \hat{\sigma}^{\mathrm{LO}}(z) + \alpha_s \hat{\sigma}^{\mathrm{NLO}}(z) + \alpha_s^2 \hat{\sigma}^{\mathrm{NNLO}}(z) + \alpha_s^3 \hat{\sigma}^{\mathrm{N3LO}}(z) + \ldots$$

The gluon fusion cross section

- The lower orders of the gluon fusion cross section have been computed
  - NLO (full theory)

    [Dawson; Djouadi, Spira, Zerwas]
  - NNLO (effective theory and sub-leading top-mass corrections)

    [Harlander, Kilgore; Anastasiou, Melnikov; Ravindran, Smith, van Neerven]

fixed order only

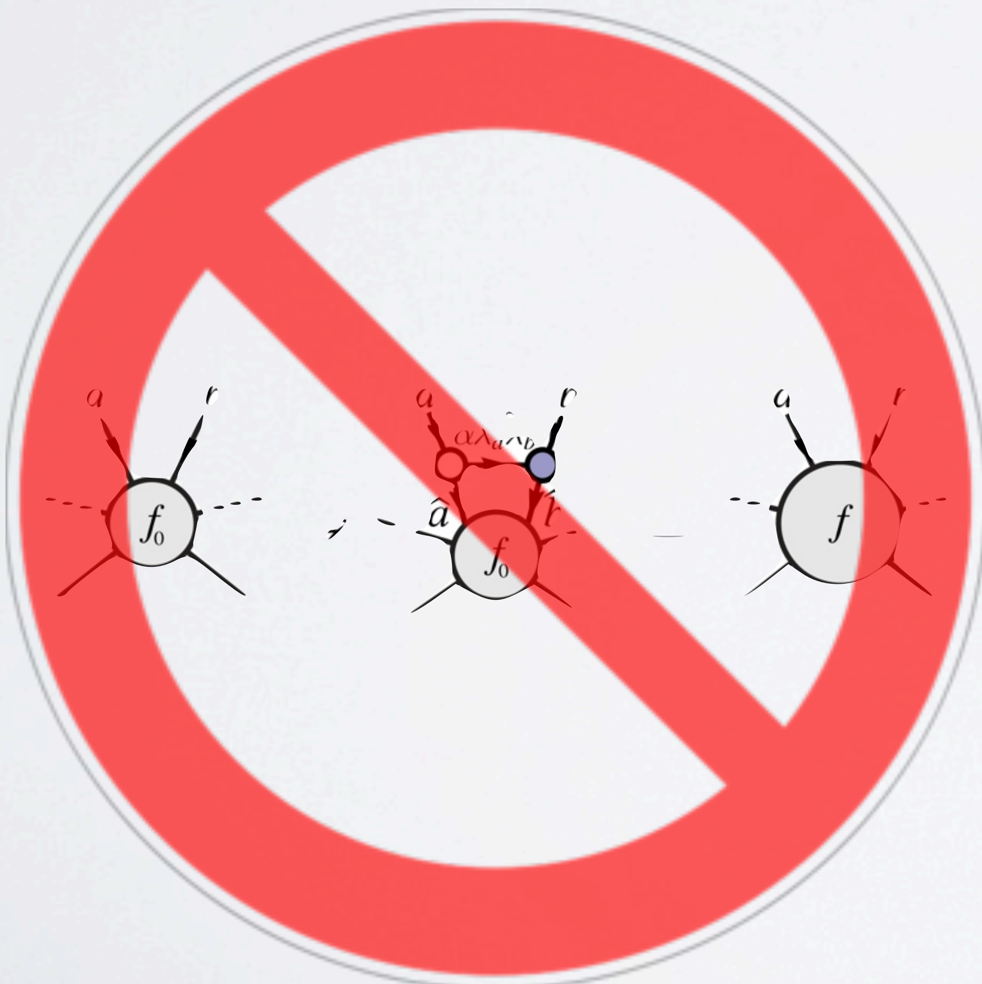
	σ [8 TeV]	$\delta\sigma$ [%]
LO	9.6 pb	~ 25%
NLO	16.7 pb	~ 20%
NNLO	19.6 pb	~ 7 - 9%
N3LO	???	~ 4 - 8%

- We want to push the calculation one order higher

- Uncharted territory in perturbation theory

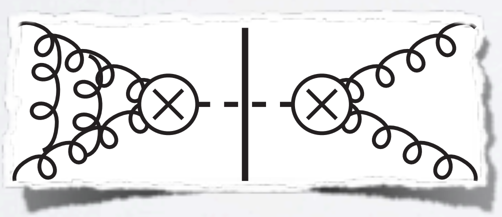
The calculation

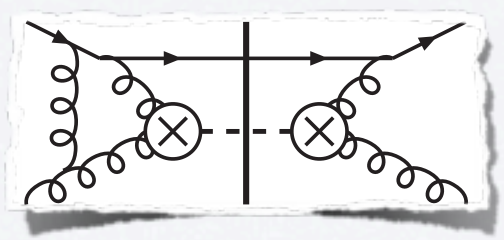
- Combination of loop corrections and real emissions computed using Feynman diagrams is the only way for analytic computations at N3LO at this point

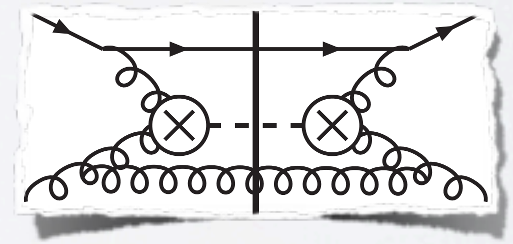


The calculation

- Combination of loop corrections and real emissions computed using Feynman diagrams is the only way for analytic computations at N3LO at this point
- Lots of Feynman diagrams
- At NNLO: ~1000 interference diagrams







The calculation

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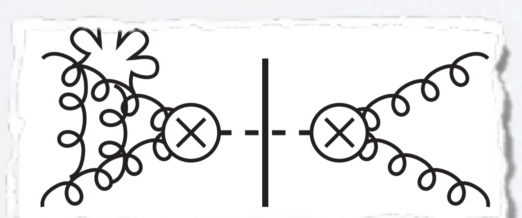
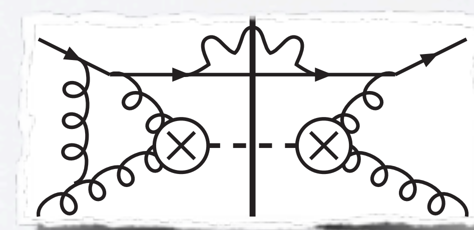
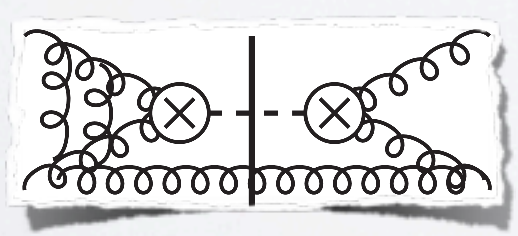
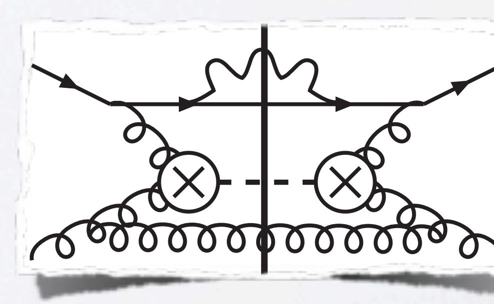
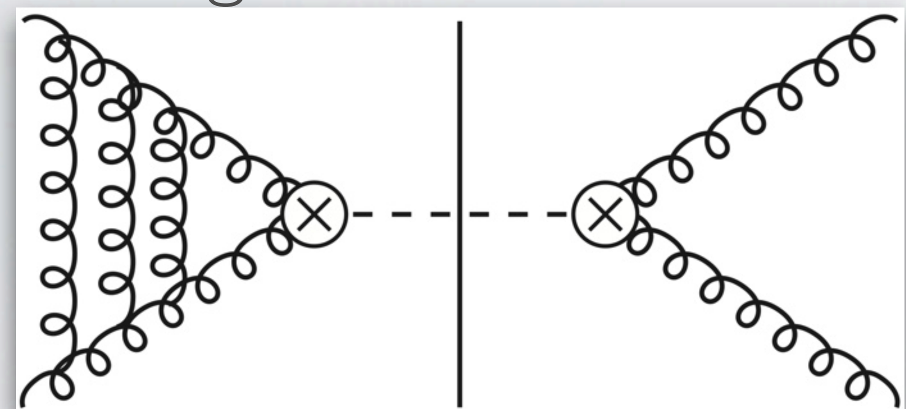
- Lots of Feynman diagrams
- At **N3LO: ~100000** interference diagrams

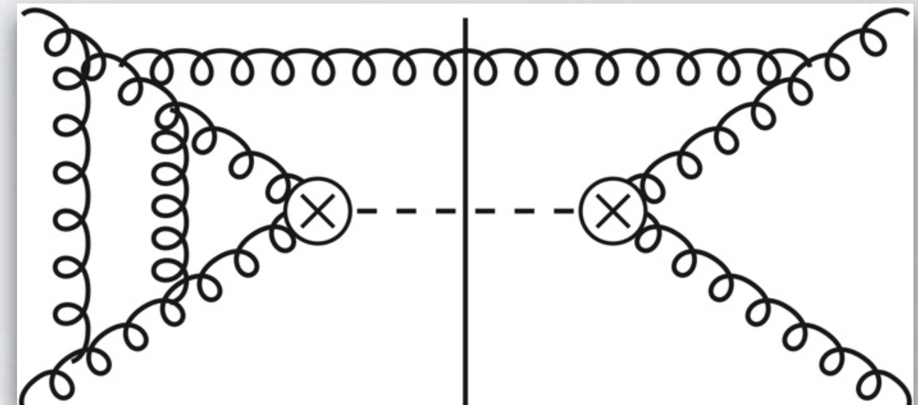





The gluon fusion cross section

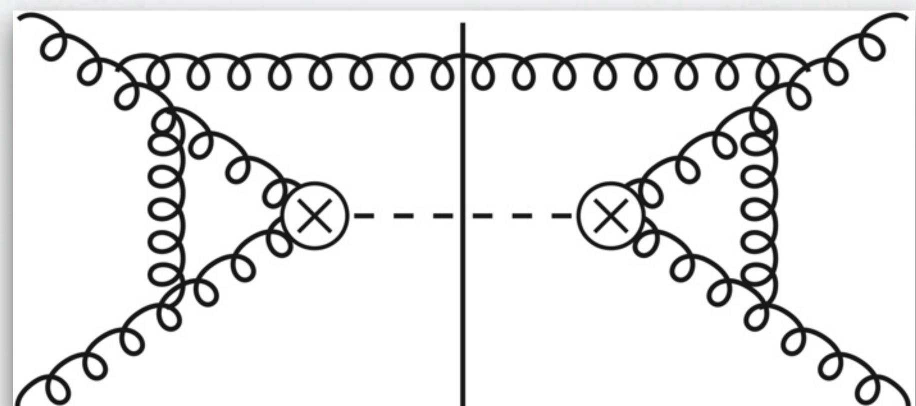
- Diagrammatic contributions at NNNLO



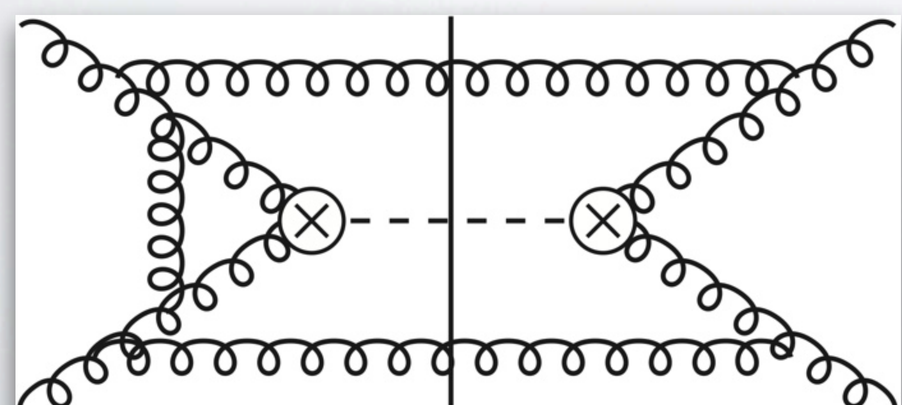
triple virtual



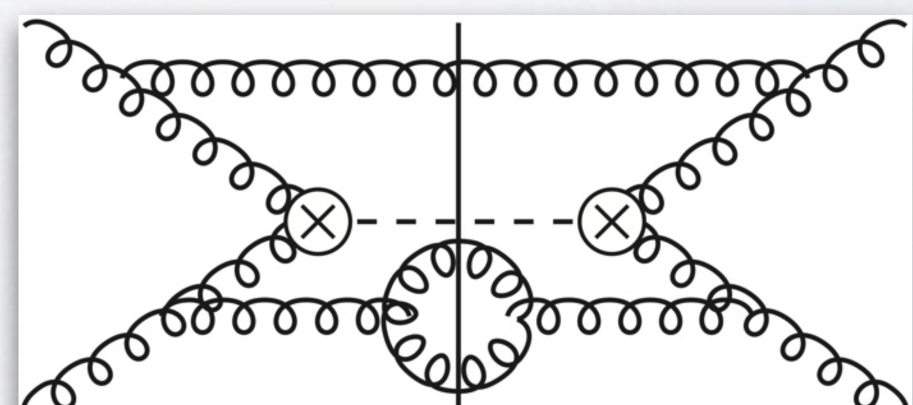
double virtual real



real virtual squared



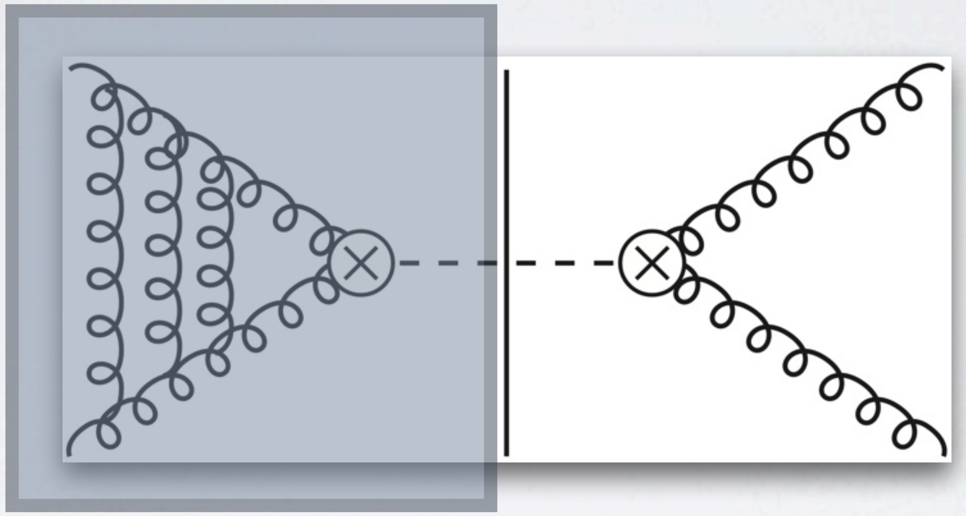
double real virtual



triple real

The triple virtual

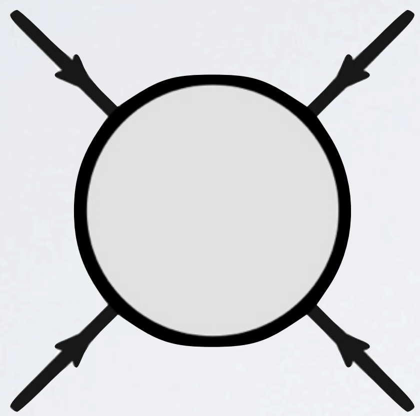
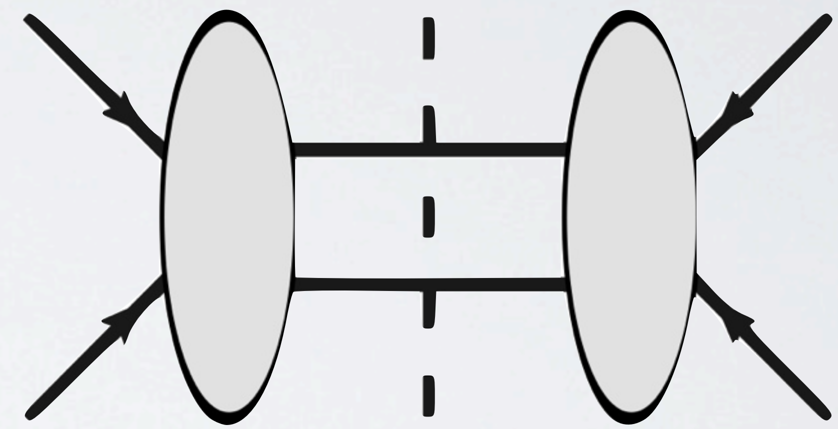
- The triple virtual is directly related to the three loop QCD form factor



- The QCD form factor is well known
  - at one loop
  - at two loops [Gonsalves; Kramer, Lampe; Gehrmann, Huber, Maitre]
  - at three loops [Baikov, Chetyrkin, Smirnov, Smirnov, Steinhauser; Gehrmann, Glover, Huber, Ikizlerli, Studerus]
- The pure loop contributions are not a problem in the calculation

Unitarity

- Optical theorem:

Im  $= \int d\Phi$ 

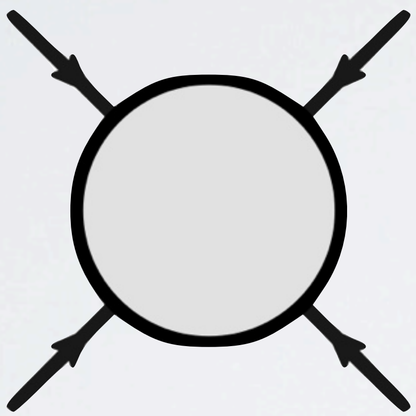
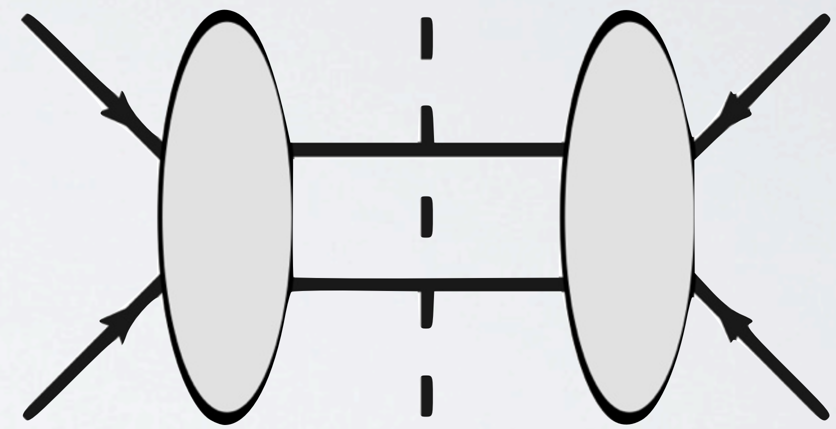
- Discontinuities of loop integrals are phase space integrals

- Discontinuities of loop integrals are given by Cutkosky's rule:

$$\frac{1}{p^2 - m^2 + i\epsilon} \to \delta^+(p^2 - m^2) = \delta(p^2 - m^2)\theta(p^0)$$

Reverse unitarity

- Optical theorem:

Im  $= \int d\Phi$ 

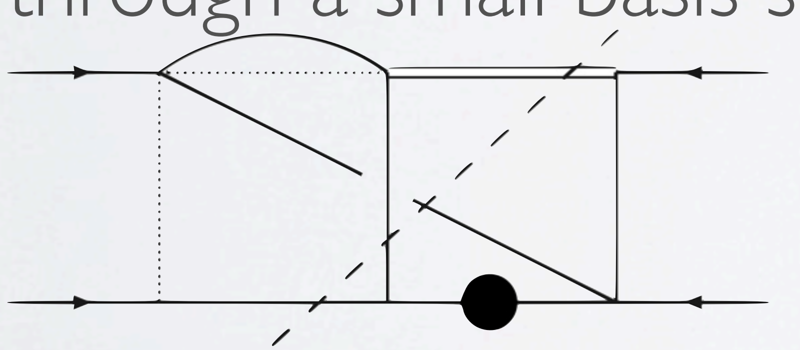
- The optical theorem can be read ‘backwards’
- This way, phase space integrals can be expressed as unitarity cuts of loop integrals [Anastasiou, Melnikov; Anastasiou, Dixon, Melnikov, Petriello]
- We can compute loop integrals with cuts instead of phase space integrals
- This makes the rich technology developed for loop integrals available

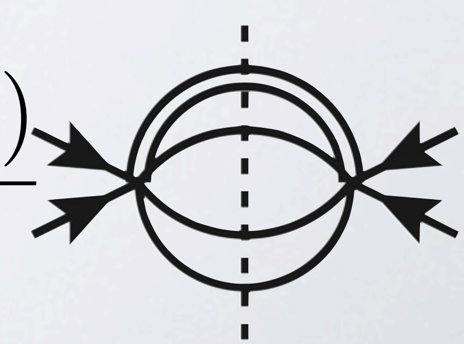
IBPs and master integrals

- Loop integrals are in general not independent but related by Integration-by-parts identities (IBPs)

- The IBPs form a system of equations for a given class of loop integrals

- The system can be solved algorithmically expressing all integrals through a small basis set of integrals (master integrals)



- $$= -\frac{(\epsilon-1)(2\epsilon-1)(3\epsilon-2)(3\epsilon-1)(6\epsilon-5)(6\epsilon-1)}{\epsilon^4(\epsilon+1)(2\epsilon-3)}$$ 

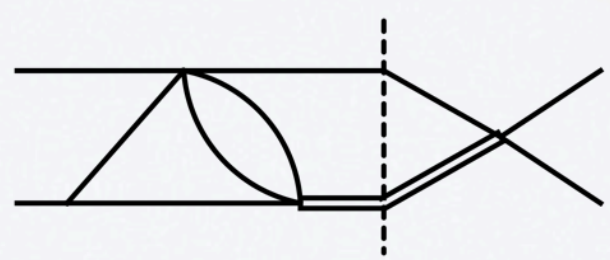
IBPs and differential equations

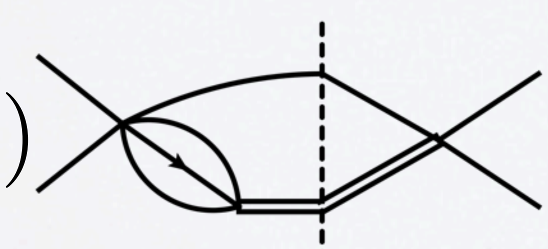
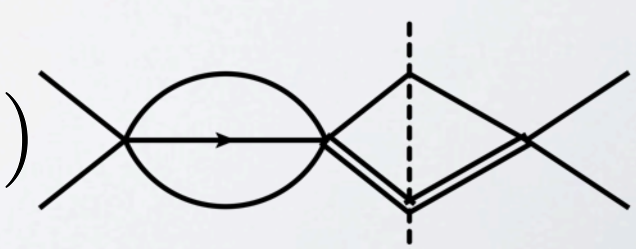
- Having access to IBP technology allows us to derive differential equations for master integrals

- The derivative of a master integral w.r.t. kinematic invariants can be expressed as a linear combination of master integrals

- Leads to a coupled system of linear differential equations for the master integrals

$$\bar{z} = 1 - z = \frac{s - m_h^2}{s}$$

$$\left[\partial_{\bar{z}} - 3\epsilon \, \mathrm{dlog}(1-\bar{z})\right]$$


- $= \epsilon \, \mathrm{dlog}(1-\bar{z})$  $-3\epsilon \, \mathrm{dlog}(1-\bar{z})$ 

Differential equations and boundaries

- Integrating the differential equations for the master integrals yields general solutions
- These general solutions need to be fixed using boundary conditions
- Natural boundary condition for the problem at

$$\bar{z} = 0 \Longleftrightarrow \hat{s} = m_h^2$$

- This corresponds to the soft or threshold limit of the process
- Higgs is produced on shell
- Any additional radiation is low energetic (soft)

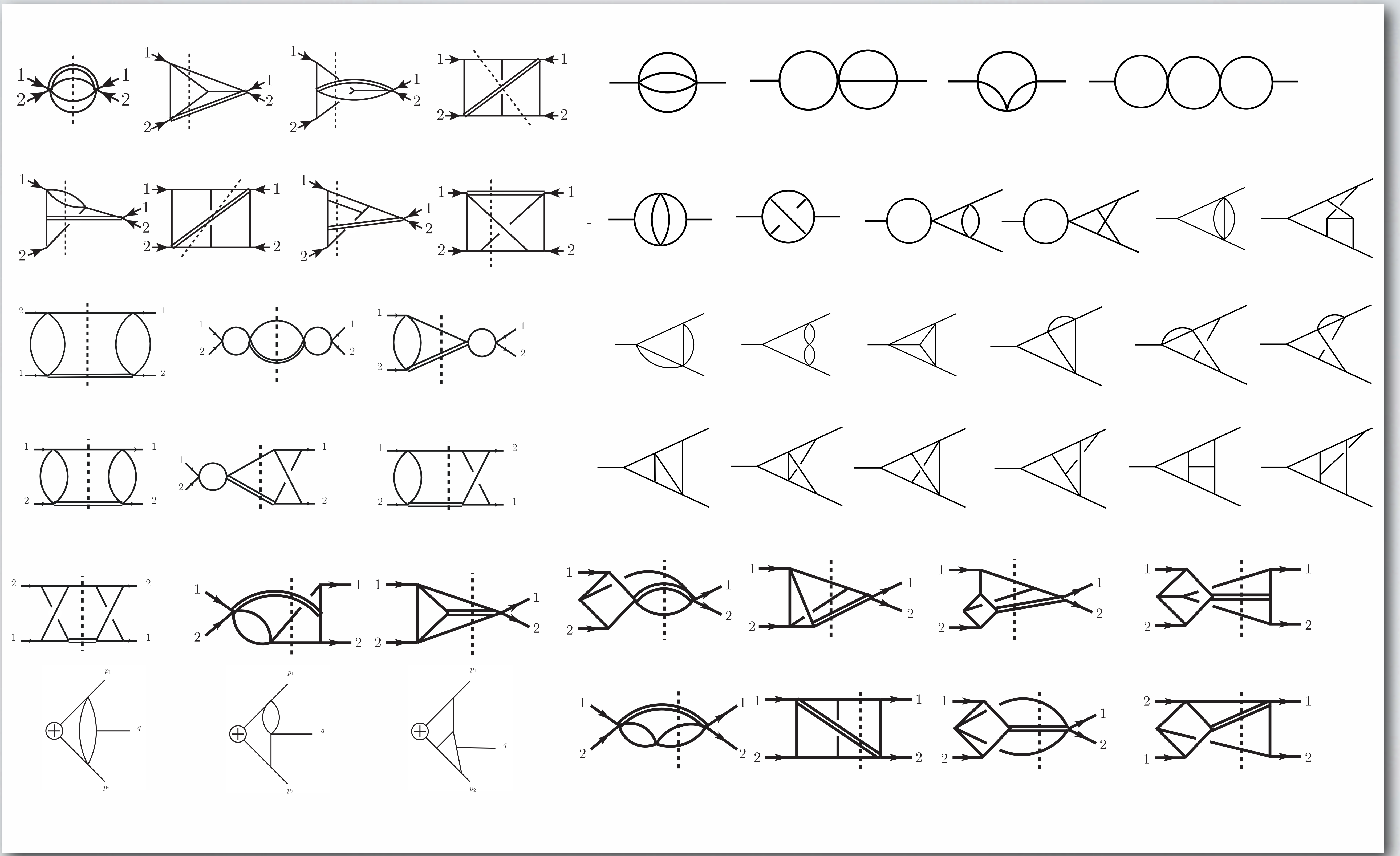
Threshold expansion

- Systematic expansion of the cross section

$$\hat{\sigma}(\bar{z}) = \sigma^{(-1)} + \sigma^{(0)} + \sigma^{(1)}\bar{z} + \dots$$

- Expansion reduces complexity of the calculation
- Reduced number of integrals that need to be computed
- First approximation of the cross section
- Most important ingredient for calculating the full result

The master integrals



The master integrals

- One of the biggest challenges of the project
- Calculation of the integrals is only possible using a variety of the most modern techniques
- Lots of inspiration from number theory
- Required development of new technologies

The master integrals

The soft-virtual approximation

- All required integrals can be computed analytically
  - 22 three-loop integrals [Baikov, Chetyrkin, Smirnov, Smirnov, Steinhauser; Gehrmann, Glover, Huber, Ikizlerli, Studerus]
  - 3 double-virtual real integrals [Duhr, Gehrmann; Li, Zhu]
  - 7 real-virtual squared integrals [Anastasiou, Duhr, FD, Herzog, Mistlberger; Kilgore]
  - 9 double-real virtual integrals [Anastasiou, Duhr, FD, Herzog, Mistlberger; Li, von Manteufel, Schabinger, Zhu]
  - 8 triple real integrals [Anastasiou, Duhr, FD, Mistlberger]

- Additionally
  - three-loop splitting functions [Moch, Vogt, Vermaseren]
  - three-loop beta functions [Tarasov, Vladimirov, Zharkov; Larin, Vermaseren]
  - three-loop Wilson coefficient [Chetyrkin, Kniehl, Steinhauser; Schroder, Steinhauser; Chetyrkin, Kuhn, Sturm]

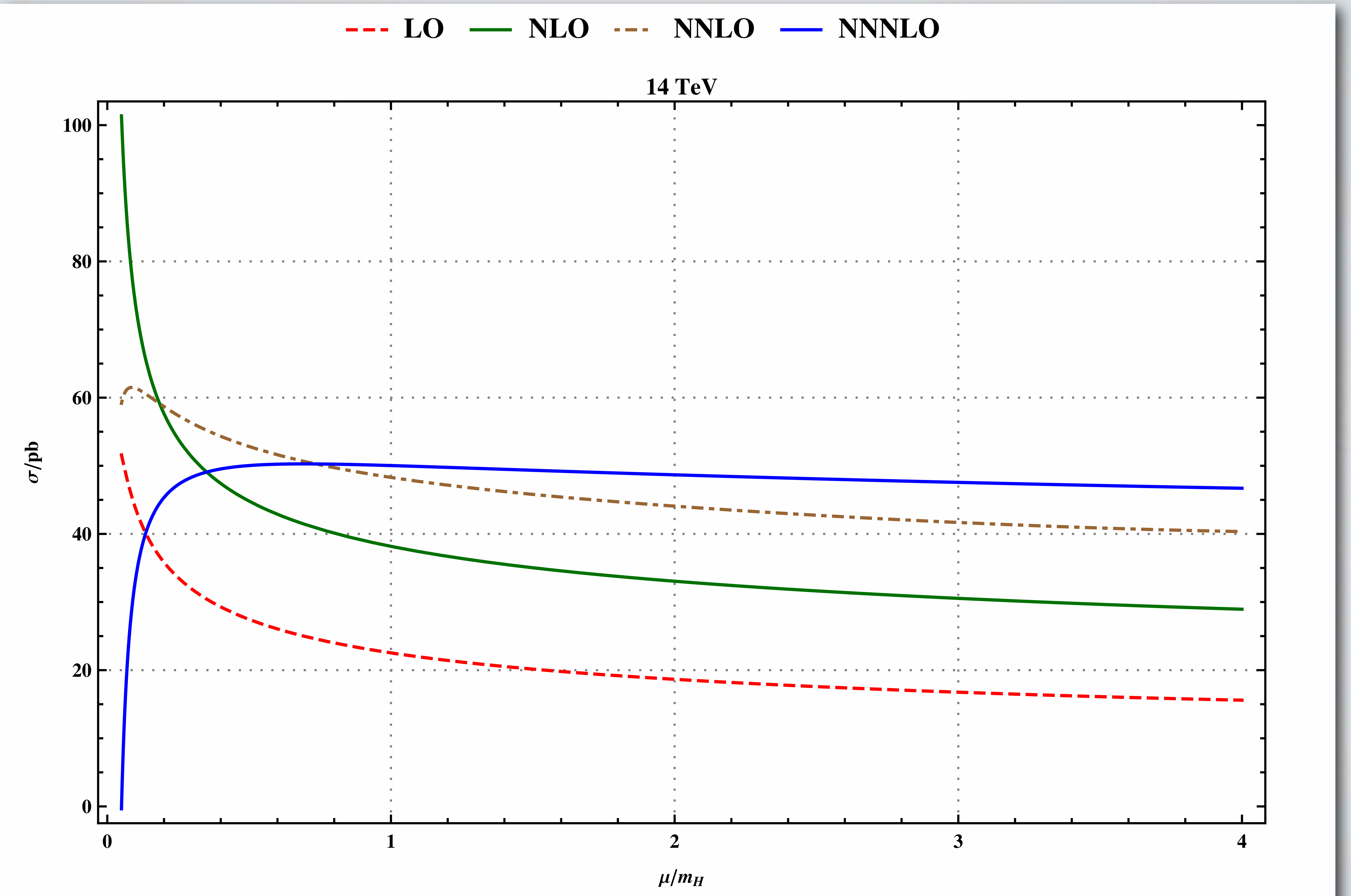
The soft-virtual cross section at N3LO

$$\begin{aligned}
\hat{\eta}^{(3)}(z) &= \delta(1-z)\Bigg\{C_A^3\left(-\frac{2003}{48}\zeta_6+\frac{413}{6}\zeta_3^2-\frac{7579}{144}\zeta_5+\frac{979}{24}\zeta_2\zeta_3-\frac{15257}{864}\zeta_4-\frac{819}{16}\zeta_3+\frac{16151}{1296}\zeta_2+\frac{215131}{5184}\right)\\
&\quad+N_F\left[C_A^2\left(\frac{869}{72}\zeta_5-\frac{125}{12}\zeta_3\zeta_2+\frac{2629}{432}\zeta_4+\frac{1231}{216}\zeta_3-\frac{70}{81}\zeta_2-\frac{98059}{5184}\right)\right.\\
&\quad+N_F^2\left[C_A\left(-\frac{19}{36}\zeta_4+\frac{43}{108}\zeta_3-\frac{133}{324}\zeta_2+\frac{2515}{1728}\right)+C_F\left(-\frac{1}{36}\zeta_4-\frac{7}{6}\zeta_3-\frac{23}{72}\zeta_2+\frac{4481}{2592}\right)\right]\Bigg\}\\
&+\left[\frac{1}{1-z}\right]_+\Bigg\{C_A^3\left(186\zeta_5-\frac{725}{6}\zeta_3\zeta_2+\frac{253}{24}\zeta_4+\frac{8941}{108}\zeta_3+\frac{8563}{324}\zeta_2-\frac{297029}{23328}\right)+N_F^2C_A\left(\frac{5}{27}\zeta_3+\frac{10}{27}\zeta_2-\frac{58}{729}\right)\\
&\quad+N_F\left[C_A^2\left(-\frac{17}{12}\zeta_4-\frac{475}{36}\zeta_3-\frac{2173}{324}\zeta_2+\frac{31313}{11664}\right)+C_AC_F\left(-\frac{1}{2}\zeta_4-\frac{19}{18}\zeta_3-\frac{1}{2}\zeta_2+\frac{1711}{864}\right)\right]\Bigg\}\\
&+\left[\frac{\log(1-z)}{1-z}\right]_+\Bigg\{C_A^3\left(-77\zeta_4-\frac{352}{3}\zeta_3-\frac{152}{3}\zeta_2+\frac{30569}{648}\right)+N_F^2C_A\left(-\frac{4}{9}\zeta_2+\frac{25}{81}\right)\\
&\quad+N_F\left[C_A^2\left(\frac{46}{3}\zeta_3+\frac{94}{9}\zeta_2-\frac{4211}{324}\right)+C_AC_F\left(6\zeta_3-\frac{63}{8}\right)\right]\Bigg\}\\
&+\left[\frac{\log^2(1-z)}{1-z}\right]_+\Bigg\{C_A^3\left(181\zeta_3+\frac{187}{3}\zeta_2-\frac{1051}{27}\right)+N_F\left[C_A^2\left(-\frac{34}{3}\zeta_2+\frac{457}{54}\right)+\frac{1}{2}C_AC_F\right]-\frac{10}{27}N_F^2C_A\Bigg\}\\
&+\left[\frac{\log^3(1-z)}{1-z}\right]_+\Bigg\{C_A^3\left(-56\zeta_2+\frac{925}{27}\right)-\frac{164}{27}N_FC_A^2+\frac{4}{27}N_F^2C_A\Bigg\}\\
&+\left[\frac{\log^4(1-z)}{1-z}\right]_+\left(\frac{20}{9}N_FC_A^2-\frac{110}{9}C_A^3\right)+\left[\frac{\log^5(1-z)}{1-z}\right]_+8C_A^3.
\end{aligned}$$

The soft-virtual cross section at N3LO

- This result contains the full three-correction and all corrections coming from the emission of up to three soft gluons
- How did we make sure that it is correct?
  - We observe the extremely intricate cancellation of six poles in dimensional regularization
  - The plus distribution terms agree with a calculation by Moch and Vogt
  - All master integrals were calculated analytically and cross checked numerically
  - We performed internal independent calculations for all pieces and some contributions have been calculated and confirmed by other groups as well

The soft-virtual cross section at N3LO



Caveat

- The soft-virtual cross section is only the first term of the expansion

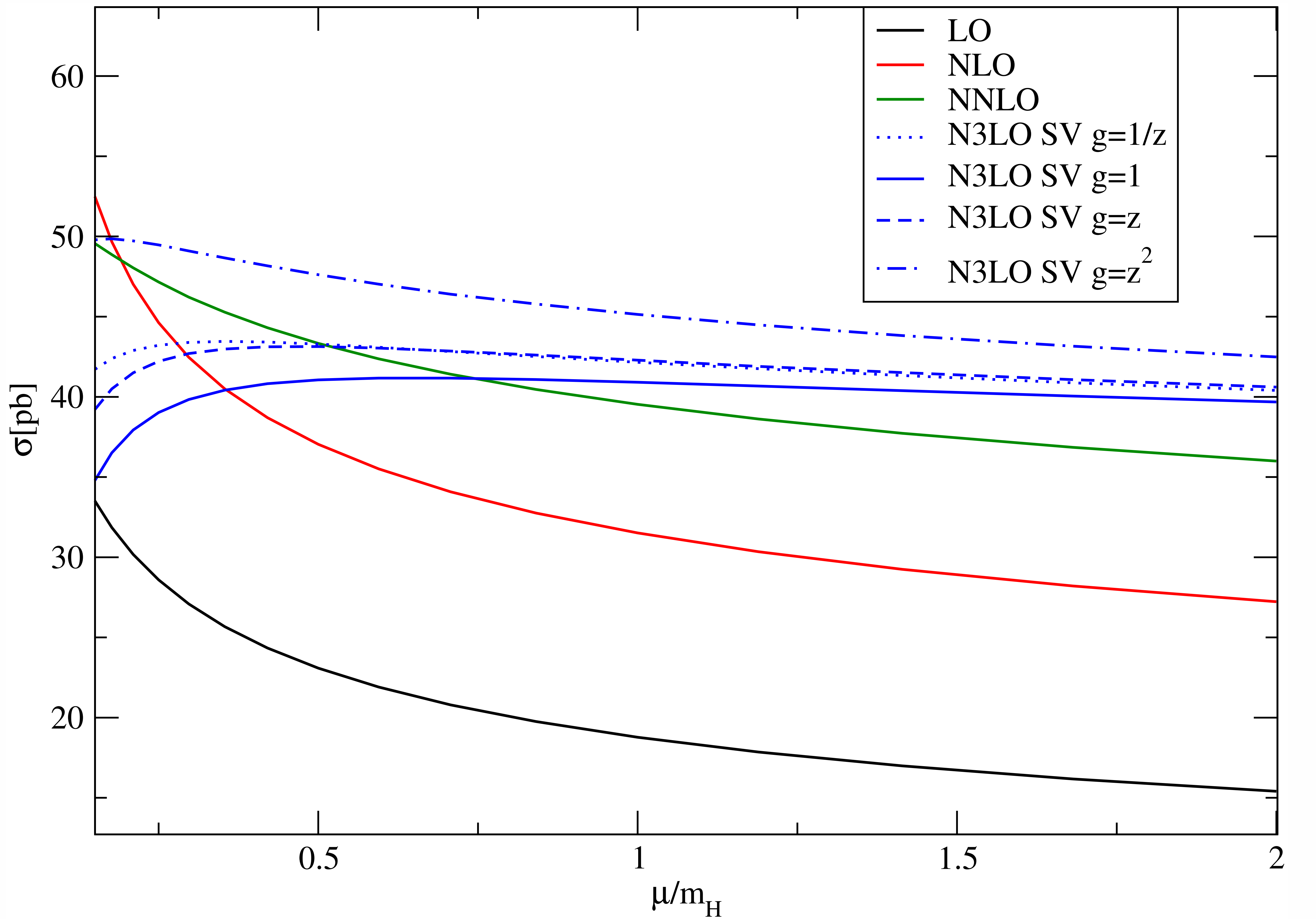
- Soft-virtual term is ambiguous

$$\sigma = \int dx_1 dx_2 \mathrm{pdf}(x_1)\mathrm{pdf}(x_2)[zg(z)] \left[\frac{\hat{\sigma}(z)}{zg(z)}\right]_{\mathrm{threshold}}$$

- We can choose any $g(z)$ as long as $\lim_{z\to 1} g(z) = 1$

$g(z)$	1	z	z^2	$1/z$
$\frac{\delta\sigma^{N3LO}}{\sigma^{LO}}$	-2.27%	8.19%	30.16%	7.73%

The soft approximation at N3LO



Outlook & Conclusion

- We have completed the first calculation of the Higgs boson cross section at N3LO in the soft-virtual approximation

- Calculation of more terms in the expansion in progress

- More terms will allow for phenomenologically meaningful predictions

- Will result in an updated prediction for the Higgs cross section at N3LO

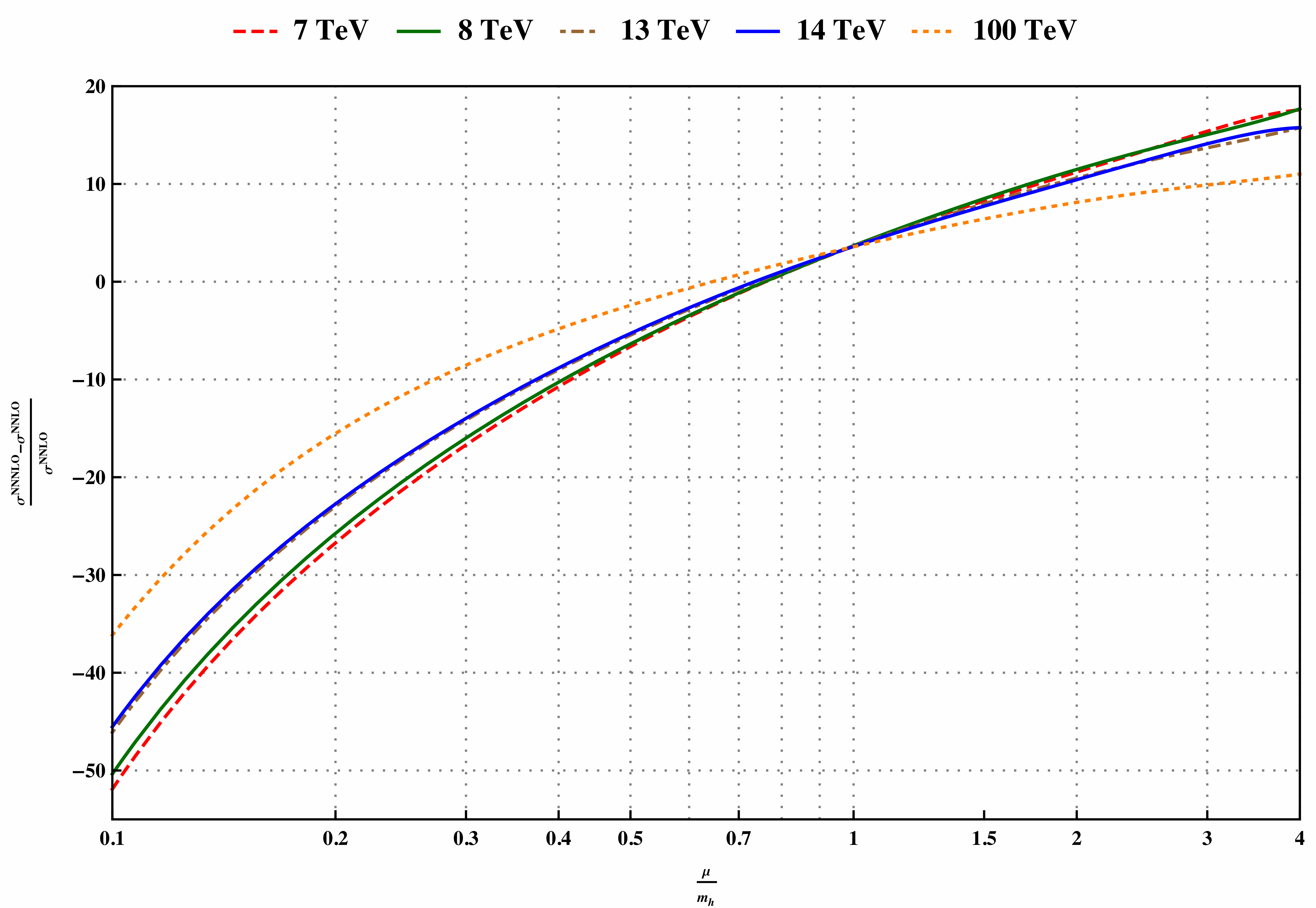
Outlook & Conclusion

- The soft-virtual term provides boundary conditions to the full kinematic solution
- Calculation of the full kinematic result in progress
- Our methods open opportunities for further calculations: Drell-Yan, SuSy Higgs, eventually 2 to 2 processes

Thank you for your attention

Backup slides

The soft-virtual cross section at N3LO



Threshold expansion

How fast is the convergence?